Solutions

In-Class Activities

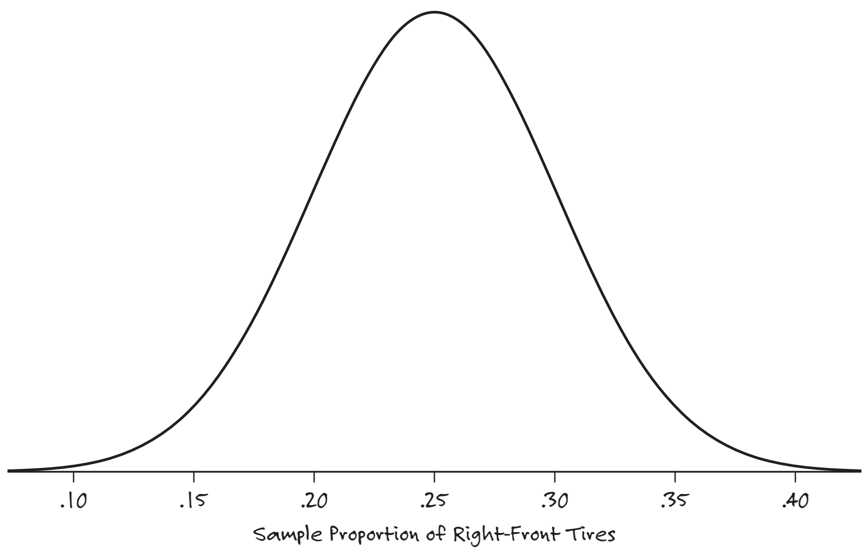
Activity 17-1: Flat Tires

a. 1/4 or .25

b. This value is a parameter because it represents the overall process, not simply results that you observed, and is represented by π.

c. $\pi > .25$

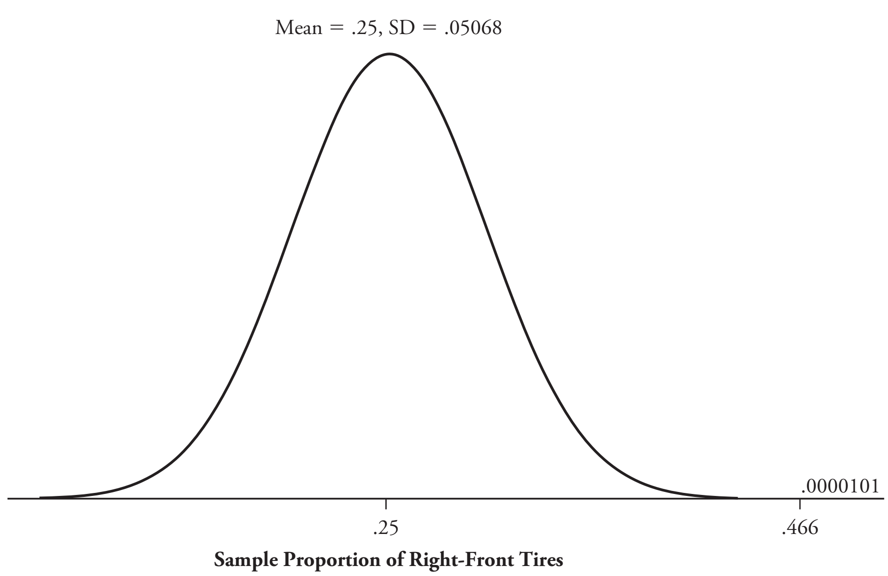
d. The sampling distribution of the sample proportion will be approximately normal, with mean equal to .25, and standard deviation equal to $\sqrt{.25(1 - .25)/73} =$ .0507. Here is a sketch of the sampling distribution:



e. The conditions necessary for the CLT to be valid are that $n\pi \geq 10$ and $n(1 - \pi) \geq 10$. These conditions are met ($73(.25) > 10$ and $73(.75) > 10$). However, you also need to believe that the sample is representative of the larger population process. This is less clear, but there may not be any reason to believe that this professor's class would behave substantially differently on this issue than college students in general, or you may want to think more carefully about how you define the population in this activity (e.g., students of similar age and major).

f. To determine the sample proportion, you calculate $\hat{p} = \frac{34}{73} = .466$. Yes, this sample proportion is greater than 1/4.

g. The following graph displays the shaded area:



For the z-score, you calculate

$$z = \frac{.466 - .25}{.0507} = 4.26$$

p-value $= \Pr(Z > 4.26) < 0.0002$

h. This sample result would be surprising if there were nothing special about the right-front tire. With a *p*-value of approximately zero, you would consider this sample result so surprising that you would conclude the right-front tire would be chosen by more than one-fourth of the population.

i. Let π represent the proportion of the population who will select the right-front tire when asked this "flat tire" question.

j. H_0: $\pi = .25$

k. H_a: $\pi > .25$

l. The test statistic is $z = \dfrac{.466 - .25}{\sqrt{.25(1 - .25)/73}} = \dfrac{.466 - .5}{.0507} = 4.26$.

m. The *p*-value $< .0002$.

n. Yes, this probability suggests that it is very unlikely for 34 or more of 73 randomly selected students to choose the right-front tire if one-fourth of the population would choose the right-front tire. Such a sample result would occur in less than 1% of random samples if $\pi = .25$.

o. Calculate $73(.25) = 18.25 > 10$ and $73(.75) = 54.75 > 10$, so this condition is met. You are not certain this is a simple random sample from the population of interest, but it is likely to be a representative sample of introductory statistics students.

p. You have found very strong statistical evidence that *introductory statistics students* tend to choose the right-front tire more than one-fourth of the time.

Activity 17-2: Flat Tires

a. Define parameter of interest: Let π represent the proportion of all introductory statistics students who will select the right-front tire when asked this "flat tire" question.

H_0: $\pi = .25$

H_a: $\pi > .25$

Check technical conditions: $74(.25) = 18.5 > 10$ and $74(.75) = 55.5 > 10$, so this condition is met. You are not certain this is a simple random sample from the population of interest, but it is likely to be a representative sample of introductory statistics students.

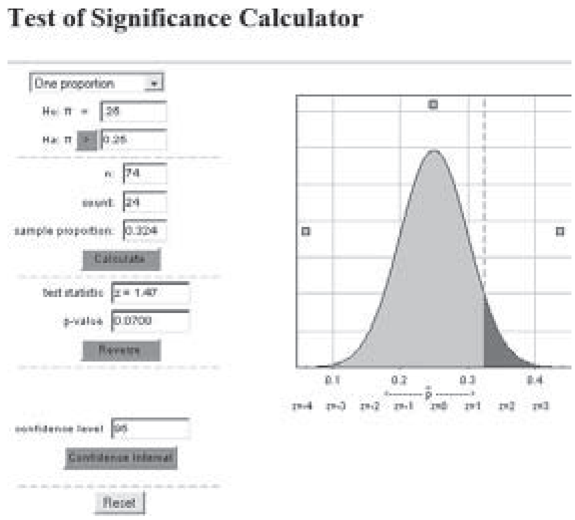
Test statistic: $z = \dfrac{.324 - .25}{\sqrt{\dfrac{(.25)(.75)}{73}}} = 1.47$

p-value $= \Pr(Z > 1.47) = .0708$

Test decision: At $\alpha = .05$ level, do not reject H_0 (*p*-value $= .0708 > .05$).

Conclusion in context: You do not have sufficient statistical evidence at the 5% level (though you do at the 10% level) to conclude that the proportion of statistics students who will choose the right-front tire is greater than one-fourth. You will continue to believe that the right-front tire is no more likely to be chosen than any other tire.

b. The following graph confirms calculation of the test statistic and p-value:



Activity 17-3: Racquet Spinning

a. Population parameter of interest: Let π represent the proportion of times a spun tennis racquet will land "up."

b. H_0: $\pi = .5$ (A spun tennis racquet will land "up" half the time.)

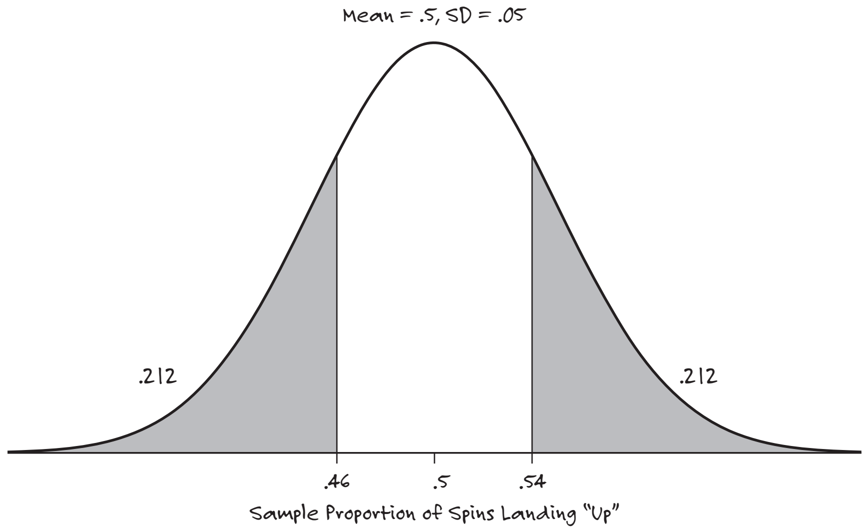
H_a: $\pi \neq .5$ (A spun tennis racquet will not land "up" half the time.)

c. Technical conditions: As long as there was nothing unusual about the spinning process, you will consider these data a "random" sample, and $100(.5) = 100(1 - .5) = 50 > 10$, so the technical conditions for the validity of this test procedure are satisfied.

d. The test statistic is $z = \dfrac{.46 - .5}{\sqrt{\dfrac{(.5)(.5)}{100}}} = -0.80$.

e. p-value $= 2 \times \Pr(Z < -0.80) = 2 \times .2119 = .4238$

Here is a sketch of the standard normal curve with the *p*-value shaded:



f. This *p*-value is very large (.4238 > .05). Therefore, do not reject the null hypothesis at the .05 significance level.

g. Conclusion in context: You do not have statistical evidence that would allow you to conclude that a spun tennis racquet will fail to land "up" half the time.

Activity 17-4: Flat Tires

a. You need more information in order to decide whether this constitutes strong evidence that the right-front tire would be chosen more than one-quarter of the time in the long run. You need to know the number of people surveyed (the sample size).

b. See values in table following part c.

c. Here is the completed table:

Sample Size	# "right-front"	$\hat{p}$	*z* statistic	*p*-value	$\alpha = .10$?	$\alpha = .05$?	$\alpha = .01$?	$\alpha = .001$?
50	15	.30	0.82	.207	No	No	No	No
100	30	.30	1.15	.124	No	No	No	No
150	45	.30	1.41	.079	Yes	No	No	No
250	75	.30	1.83	.034	Yes	Yes	No	No
500	150	.30	2.58	.005	Yes	Yes	Yes	No
1000	300	.30	3.65	.000	Yes	Yes	Yes	Yes

d. When the sample size is small, a sample result of .30 is not statistically significant at any level. But, as the sample size increases, this result becomes more significant—meaning that it becomes more unlikely that you would obtain a sample result of .30 (or more extreme) if the population parameter is actually .25 as you use larger and larger samples.

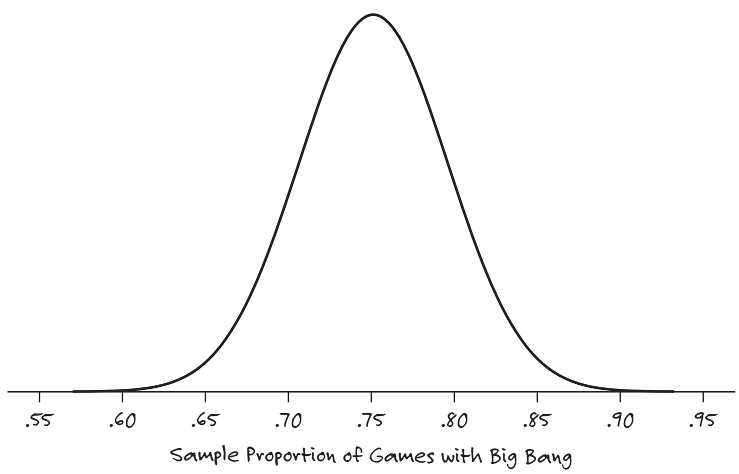
Activity 17-5: Baseball "Big Bang"

a. The null hypothesis is that the proportion of all major-league baseball games that contain a big bang is three-fourths. In symbols, the null hypothesis is H_0: $\pi = .75$.

b. The alternative hypothesis is that less than three-fourths of all major-league baseball games contain a big bang. In symbols, the alternative hypothesis is H_a: $\pi < .75$.

c. A week of games was randomly selected. Although this does not constitute a simple random sample of all games, you do hope it is representative of the scores in such games. The CLT applies here because $95(.75) = 71.25$ is greater than 10, and $95(.25) = 23.75$ is also greater than 10. According to the CLT, the sample proportion would vary approximately normally, with mean .75 and standard deviation equal to

$$\sqrt{\frac{(.75)(.25)}{95}} \approx .0444$$



d. The sample proportion of games in which a big bang occurred is

$$\hat{p} = \frac{47}{95} \approx .495$$

e. Yes, this sample proportion is less than .75, as Marilyn conjectured.

f. The test statistic is

$$z = \frac{.495 - .75}{\sqrt{\frac{(.75)(.25)}{95}}} \approx \frac{.495 - .75}{.0444} \approx -5.74.$$

This test statistic says that the observed sample result is almost six standard deviations below what the grandfather conjectured. This z-score is way off the chart in Table II, indicating that the p-value is virtually zero ($< .0002$).

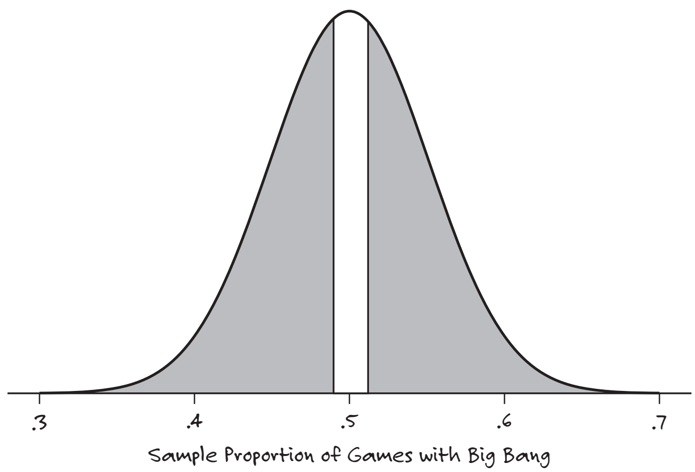
g. Yes, this very small p-value indicates that the sample data provide extremely strong evidence against the grandfather's claim. There is extremely strong evidence that less than 75% of all major-league baseball games contain a big bang. The null (grandfather's) hypothesis would be rejected at the $\alpha = .01$ level.

h. The hypotheses for testing Marilyn's claim are H_0: $\pi = .5$ vs. H_a: $\pi \neq .5$.

i. The test statistic is

$$z = \frac{.495 - .5}{\sqrt{\frac{(.5)(.5)}{95}}} \approx \frac{.495 - .5}{.0513} \approx -0.10$$

The p-value is $2(.4602) = .9204$.



j. This p-value is not small at all, suggesting that the sample data are quite consistent with Marilyn's hypothesis that half of all games contain a big bang. The sample data provide no reason to doubt Marilyn's hypothesized value for π.

k. A 95% confidence interval for π (the population proportion of games that contain a big bang) is given by

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

with $z^* = 1.96$, which is

$$.495 \pm 1.96\sqrt{\frac{(.495)(.505)}{95}}$$

which is $.495 \pm .101$, which is the interval from .394 to .596. Therefore, you are 95% confident that between 39.4% and 59.6% of all major-league baseball games contain a big bang. The grandfather's claim (75%) is not within this interval or even close to it, which explains why it was so soundly rejected. Marilyn's conjecture (50%) is well within this interval of plausible values, which is consistent with it not being rejected.